

Mathematics Teachers' Use of Mathematical Descriptions, Explanations and Justifications While Teaching Function Concept: The Case of Samet

Matematik Öğretmenlerinin Matematiksel Tanımlamaları, Açıklamaları ve Doğrulamaları Kullanımı: Samet Örneği

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Received: 02 November 2018

Research Article

Accepted: 07 February 2019

ABSTRACT: It is important for teachers to design and use mathematically accurate descriptions, explanations and justifications that are comprehensible and useful for students in the context of reflecting their mathematical knowledge for teaching. The purpose of this study is to examine a mathematics teacher's mathematical knowledge for teaching function concept by investigating his use of mathematical descriptions, explanations and justifications. The study was conducted as a descriptive case study. The participant in the study was one mathematics teacher (Samet) who volunteered to join the research. Data was collected via observing and recording the teacher's teaching of the function concept and survey of function concept. The results of the study revealed that the teacher mostly used mathematical descriptions in his teaching. This was followed by mathematical explanations, and mathematical justifications. The teacher's use of mathematical descriptions, explanations and justifications and his sufficiency at using these varied according to cases. Results indicated some deficiencies in the teacher's mathematical knowledge for teaching.

Keywords: mathematical knowledge for teaching, teaching function concept, mathematical descriptions, mathematical explanations, mathematical justifications.

ÖZ: Öğretmenlerin, öğretmek için matematik bilgilerini yansıtma ve öğrenciler için anlaşılır ve kullanışlı ve matematiksel olarak doğru tanımlamaları, açıklamaları ve doğrulamaları tasarlamaları ve kullanmaları önemlidir. Bu çalışmanın amacı, bir matematik öğretmenin fonksiyon kavramının öğretiminde matematiksel tanımlamaları, açıklamaları ve doğrulamaları kullanımını araştırarak öğretmek için matematik bilgisini incelemektir. Çalışma, tanımlayıcı bir durum çalışması olarak gerçekleştirilmiştir. Araştırmanın katılımcısı araştırmaya katılmak için gönüllü olan bir matematik öğretmenidir (Samet). Veriler, öğretmenin fonksiyon kavramı öğretiminin gözlemlenmesi ve kaydedilmesi ve fonksiyon kavramı anketi ile toplanmıştır. Araştırmanın sonuçları, öğretmenin öğretiminde çoğunlukla matematiksel tanımlamaları kullandığını ortaya koymuştur. Bunu matematiksel açıklamalar ve matematiksel doğrulamalar takip etmiştir. Öğretmenin matematiksel tanımlamaları, açıklamaları ve doğrulamaları kullanımı ve bunları kullanmadaki yeterliliği farklı durumlara göre değişiklik göstermiştir. Sonuçlar, öğretmenin öğretmek için matematik bilgisindeki bazı eksiklikleri olduğunu göstermiştir.

Anahtar kelimeler: öğretmek için matematik bilgisi, fonksiyon kavramı öğretimi, matematiksel tanımlamalar, matematiksel açıklamalar, matematiksel doğrulamalar.

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Citation Information

Tataroğlu-Taşdan, B. (2019). Mathematics teachers' use of mathematical descriptions, explanations and justifications while teaching function concept: The case of Samet. *Kuramsal Eğitim Bilim Dergisi [Journal of Theoretical Educational Science]*, 12(4), 1284-1305.

Introduction

It is known that the improvement of mathematics education for all students requires effective mathematics teaching in all classrooms (National Council of Teachers of Mathematics [NCTM], 2000). To achieve effective teaching, mathematics teachers need to have various knowledge and skills. Although teaching mathematics well is a complex endeavour, and there are no recipes for helping all students learn or for helping all teachers become effective (NCTM, 2000), research results help teachers and researchers to achieve this by presenting pedagogical models of teacher knowledge (An, Kulm, & Wu, 2004; Ball, Thames, & Phelps, 2008; Fennema & Franke, 1992; Grossman, 1990; Magnusson, Krajcik, & Borko, 1999; Shulman, 1986, 1987).

Pedagogical content knowledge (PCK), one of the most important conceptions of teacher knowledge, was first introduced by Shulman (1986, 1987). Shulman (1987) defined PCK as “the special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding” (p. 8). After Shulman (1986, 1987) proposed the notion of PCK, many researchers studying the field of teacher education used PCK for developing teacher knowledge models and frameworks. They re-conceptualized PCK by implementing it in various disciplines. Within the context of mathematics education, Deborah Ball and her colleagues studied how mathematics teachers carry out the work of teaching mathematics. Then Ball et al. (2008) stated that teaching for understanding requires special mathematical knowledge for teaching (MKT) and proposed the MKT model for effective mathematics teaching, building on the concept of PCK (Shulman, 1986, 1987). By MKT, they meant the mathematical knowledge needed to carry out the work of teaching mathematics. They also highlighted that their definition begins with teaching, not teachers (Ball et al., 2008). Ball et al. (2008) see teaching as everything that teachers must do to support the learning of their students. Similarly, Hill et al. (2008) stated that by MKT they mean not only the mathematical knowledge common to individuals working in diverse professions, but also the subject matter knowledge that supports that teaching, for example, why and how specific mathematical procedures work, how best to define a mathematical term for a particular grade level, and the types of errors students are likely to make with particular content.

When we look at the literature, we see that a group of researchers (Hill et al., 2008; Snider, 2016) that used MKT in their research have investigated the relationship between MKT's reflections on classroom practices (how the teachers implement MKT in teaching) and the quality of mathematics teaching. It should be noted that examining the quality of teachers' knowledge itself is only a step in examining the quality of mathematics teaching and learning that mathematics educators are striving to achieve (Ball & Bass, 2002). In their study, Hill et al. (2008) related teachers' mathematical knowledge to the quality of their classroom work and determined the dynamics of knowledge used in teaching by examining the relationship between teachers' MKT and the mathematical quality of their instruction. Then, the Learning Mathematics for Teaching [LMT] Project (2011) team that was consisted of Hill and her colleagues described the framework and instrument for measuring the mathematical quality of mathematics instruction in detail. They created a set of constructs and codes which capture key mathematical events in classrooms while describing the mathematical

quality of instruction. In another study, Hill (2010) examined elementary school teachers' mathematical knowledge for teaching and the relationship between such knowledge and teacher characteristics.

There are several other studies examining teachers' mathematical knowledge and mathematics instruction (Ball, 1990; Charalambous, 2010; Charalambous & Hill, 2012; Cohen, 1990; Heaton, 1992; Kersting, Givvin, Thompson, Santagata, & Stigler, 2012; Lloyd & Wilson, 1998; Snider, 2016; Steele & Rogers, 2012; Wilson, 1990). In these studies, one or more mathematics teachers' teaching practice were examined concentrating on different components like students learning, selecting and implementing mathematical tasks, applying mathematics in real-life situations, curriculum materials, providing proof and national educational policies. It is important in the studies that teacher knowledge and teaching practice are examined together. Although there are studies investigating which MKT components are used when the teacher uses descriptions, explanations and/or justifications in the class, it is possible to say that they are still limited. It is thought that increasing these researches may provide important results in terms of MKT especially in terms of teaching function concept. As Snider (2016) stated, there has been a need for investigating how teachers draw on knowledge when they enact teaching practices. The purpose of this study is to examine a mathematics teacher's teaching in terms of using mathematical descriptions, explanations and justifications to infer his MKT for function concept. In other words, by following the suggestions provided by existing teacher education literature, the aim of the research is to determine how one mathematics teacher (who represents a little sample of Turkish mathematics teachers) reflected his MKT while teaching the function concept. Even though the aim of the study is not to generalize about the teachers' knowledge and reflections of this knowledge, some inferences and implications could be obtained regarding Turkish mathematics teachers' situations. The research question is as follows:

- How does a mathematics teacher use mathematical descriptions, explanations and justifications while teaching the function concept?

Teaching Function Concept

Function is a fundamental concept of mathematics (NCTM, 2000). It is fundamental because it forms a basis for many other mathematical concepts like limit, derivative, integral. Accordingly, in Turkish mathematics curriculum, the function concept begins in grade 9 and continues throughout the high school. Besides, functions are essential in every field of applied mathematics such as statistics, computer programming, economy (Ronau, Meyer, Crites, & Dougherty, 2014). It is important to study how the teachers teach this concept and how the students learn it. In the literature, it seen that teaching function concept has been investigated by different researchers (Aksu & Kul, 2016; Even, 1993; Even & Tirosh, 1995; Hacıömeroğlu, 2006; Hatisaru & Erbaş, 2017; Karahasan, 2010; Llinares, 2000; Nyikahadzoyi, 2015; Steele, Hillen, & Smith, 2013; Stein, Baxter, & Leinhardt, 1990; Tataroğlu-Taşdan & Yiğit-Koyunkaya, 2017; Wilson, 1992). Some examined it with a general view to PCK, while others used the MKT framework. For example, Tataroğlu-Taşdan and Yiğit-Koyunkaya (2017) examined pre-service mathematics teachers' MKT in terms of function concept and the results of their study showed that pre-service mathematics teachers had limited

knowledge regarding teaching of function concept and they had difficulties to reflect their knowledge of function concept on their teaching. In their study, Hatisaru and Erbaş (2017) examined the potential interrelationships between teachers' MKT the function concept and their students' learning outcomes of this concept. They pointed that teachers' MKT and students' learning outcomes were related to a degree, but this relationship was not straightforward. They also concluded that the teachers' knowledge influenced the quality of their instructional practices, and the instructional practices played a mediating role in student learning. Steele, Hillen and Smith (2013) dealt with the prospective and practicing teachers' learning in a teaching. They found the participants showed growth in their ability to define function, to provide examples of functions and link them to the definition, in the connections they could make between function representations, and to consider the role of definition in mathematics and the K-12 classroom.

Mathematical Descriptions, Explanations and Justifications

One of the MKT components is teacher's knowledge of content and teaching (Ball et al., 2008). As a part of knowledge of content and teaching of function concept, secondary school teachers should know the different introductions for a particular topic, sequences of exercises, explanations, representations, definitions, and examples (Nyikahadzoyi, 2015). Hill et al. (2008) pointed out that teachers without mathematical knowledge cannot provide explanations, justifications, or make careful use of representations. So, each of these components are of importance for better teaching and learning of this concept.

Mathematical descriptions, explanations and justifications, three together or separately are considered in some studies (Hill et al., 2008; Lachner & Nückles, 2015; Snider, 2016; Xenofontos & Andrews, 2017). Hill et al. (2008) examined how MKT is associated with the mathematical quality of instruction. They studied a sample of ten teachers and collected their data via pencil-and-paper assessment of MKT, videotaped lessons and interviews. They analysed the data in terms of different components including mathematical descriptions, explanations and justifications. They found a significant, strong, and positive association between levels of MKT and the mathematical quality of instruction. They also found that there were many important factors that mediate this relationship, either supporting or hindering teachers' use of knowledge in practice. Xenofontos and Andrews (2017) suggested that explanations may be a useful tool for measuring prospective teachers' knowledge, in a study where they examined first-year undergraduate teacher education students' written explanations. Lachner and Nückles (2015) investigated the impact of instructors' different knowledge bases on the quality of their instructional explanations and found that deep content knowledge helped instructors generate explanations with high process-orientation. In her thesis, Snider (2016) investigated the impact of teachers' knowledge use in practice on selecting examples and explaining, which are two foundational practices in mathematics teaching. She found that different categories of explanations and teachers' knowledge use varied by explanation type.

Since the focus of this study is use of mathematical descriptions, explanations and justifications this section deals with these terms in detail. The codes created by Hill et al. (2008) were adopted in the study. Mathematical descriptions are defined as

providing clear characterizations of the steps of a mathematical procedure or a process (Hill et al., 2008). Descriptions tell only what the steps of a mathematical procedure or a process are. A mathematical description does not necessarily address the meaning or reason for these steps (Hill et al., 2008).

Explanations are practice that occur in classrooms and are shared between a teacher and their students (Snider, 2016). Ball and Bass (2002) indicate that mathematics teachers are frequently engaged in the work of mathematical explanations. They explain mathematics; they also judge the adequacy of the explanations in textbooks, given by their students, or in mathematics resource books for teachers (Ball & Bass, 2002). According to Leinhardt (2001) providing instructional explanations is a common way to support students' understanding. So, it is important to design mathematically accurate explanations that are comprehensible and useful for students (Ball & Bass, 2002). Mathematical explanations include giving mathematical meaning to ideas or procedures, namely by giving attention to the meaning of the steps or ideas, they don't necessarily provide mathematical justification (Hill et al., 2008).

Justification is a core mathematics practice (Staples, Bartlo, & Thanheiser, 2012). Mathematical justifications include deductive reasoning about why a procedure works or why something is true or valid in general (Hill et al., 2008). In studies, justification seems to be handled together with proof, argumentation or reasoning (Cai, 2003; Chazan, 1993; Staples et al., 2012; Yackel, 2001). This study is limited by the definition made by justification Hill et al (2008).

Mathematical descriptions, explanations and justifications were explained by several examples in Learning Mathematics for Teaching (LMT) Technical Report (2006). The first example is a subtraction.

$$\begin{array}{r} 5\ 13 \\ \cancel{6}3 \\ - 28 \\ \hline 35 \end{array}$$

In the context of this example, mathematical description simply meant the recounting of the steps involved in subtraction with regrouping—cross out the 3, write 13, cross out the 6 and write 5. Subtract 8 from 13 to get 5... Mathematical explanations give mathematical meaning to ideas or procedures. In this example, the teacher (or student) might explain that the crossing-out process is really a way of re-writing the 63 as 50 and 13 ones. Re-writing in this way allows one to subtract the ones and tens column without using negative numbers. Finally, mathematical justification includes deductive reasoning about why a procedure works or why something is true or valid in general. Here, the teacher might help students determine whether this algorithm can be used to subtract any two multi-digit whole numbers where trading is required (LMT, 2006).

Method

This study was conducted as a descriptive case study. Case study research is suitable for answering questions that start with how, who and why; when the researcher has little control over events and when the focus is on a contemporary phenomenon (Yin, 2009). Creswell (2003) define case study as “researcher explores in depth a

program, an event, an activity, a process, or one or more individuals" (p. 15). In this study, it was aimed to examine a mathematics teacher's teaching in terms of using mathematical descriptions, explanations and justifications to infer his MKT for function concept, so case study was selected for research design.

Participant

The participant was one mathematics teacher who volunteered to join the research. He had 14 years of experience when the research was conducted. He was a teacher interested in new developments in mathematics teaching. He stated that he had participated in a seminar on mathematics teaching methods, a seminar on new approaches in mathematics and "Preparing the Guide to the Olympics of the Scientific and Technological Research Council of Turkey" (TÜBİTAK) activity. For this study, he was assigned the nickname Samet. Samet was working at a high school which accept the students with higher scores from national tests conducted by The Turkish Ministry of National Education relative to other schools. So, it can be said that the students in Samet's class were above a certain level of mathematical success. This difference of the school structure originated the reason for choosing this teacher as a case to examine using mathematical descriptions, explanations and justifications.

Data Collection

Data was collected via observing and recording Samet's teaching of the function concept and a survey of function concept which was filled out by Samet. Marshall and Rossman (1989) define observation as "the systematic description of events, behaviours, and artefacts in the social setting chosen for study" (p. 79). Observations enable the researcher to describe existing situations using the five senses, providing a "written photograph" of the situation under study (Erlandson, Harris, Skipper, & Allen, 1993). As mechanical recording devices usually give greater flexibility than observations done by hand (Smith, 1981), video recording was preferred in this study. Samet's teaching of the function concept was observed (via unstructured observation) for 5 lessons and recorded by a video camera. The video camera was placed behind the classroom. The researcher carried out the video recording and zoomed in on the board or the people when necessary. One digital audio recorder was also in the teacher's pocket, so that the audio recording could be referred to if there was a segment that could not be understood in the video recording. In these lessons, Samet tried to define and construct the function concept and the types of function.

In the survey of function concept, there were questions helping to determine a teacher's MKT in terms of function concept. The questions in the survey which were obtained from the literature included defining function or giving an alternative definition, giving examples for function concept, estimating the students' ideas by examining their answers to specific function questions and approaches to remove some misconceptions regarding function concept. Samet completed the survey in approximately 40 minutes in writing. Data collected via survey of function concept was used as secondary data to support the observations as well.

Data Analysis

At the beginning of the data analysis, video records were transcribed verbatim. Then the researcher read the full text and coded the transcribed lessons. The data were analysed according to the codes previously determined (Creswell, 2003). The codes called “*mathematical description, mathematical explanation and mathematical justification*” adapted from Hill et al. (2008), were used for data analysis. The descriptive analysis method was conducted as data analysis.

As the descriptive analysis required, the researcher identified the main codes in the data as described in Hill et al.’s (2008) study. The main codes were mathematical description, mathematical explanation and mathematical justification. Then sub-codes were obtained. In their study, Hill et al. (2008) separated the codes as self-produced or co-produced with students. Thus, they obtained sub-codes as “*elicits student description*” and “*elicits student explanation*” (LMT, 2011). Similarly, in this study, it was an important point to distinguish whether the teacher was doing the mathematical description, explanation and justification himself or if he wanted the students to do it. Therefore, the main sub-codes for three categories (mathematical description, mathematical explanation and mathematical justification) were emerged as “does himself” and “asks students”.

Context was important in determining which data will be fetched under which code. Although the question words such as “why” and “how” that the teacher used seem like explanation or justification, looking at these words alone did not affect which code to decide. It was decided that if the teacher was waiting for explanation or justification with the why question or description was enough for him by looking at the context. The sufficiency of the teacher’s actions was also seen significant in the analysis. Therefore, the sufficiency of the teacher’s use of mathematical descriptions, mathematical explanations and mathematical justifications were sub-categorized as sufficient and insufficient. The context was considered while deciding the sufficiency of an action. Mathematical accuracy, students’ questions or comments, teacher’s deepening of students’ thoughts were the indicators of sufficiency.

Only mathematical explanations included a different sub-code, called ‘needed but absent’. This sub-code was used for the cases which suitable conditions exist for making mathematical explanations and doing it would help students to learn but the teacher did not. Sample rows taken from the data analysis are provided in Table 1.

Table 1
A Section from Data Analysis

Transcription section		Teacher's Action	Sufficiency
Samet	First one is a function. Işıl, why is it a function?	Ask a student for mathematical explanation	
Student	Because all element in set A		Insufficient (It is understood that the student could not explain her thoughts, however the teacher did not encourage her to do)
Samet	All the elements in A have an element in B that matches. Is there a non-matched element in the definition set?	Does a mathematical explanation himself	Insufficient (He pointed on only one of the conditions of being a function, not to the other. For this reason, the explanation is insufficient.)
Stud. (together)	No.		
Samet	Not. So, relation f_1 is a function from A to B.		

To ensure validity and reliability, a second coding was realized by the (same) researcher nearly two months after the first coding, in accordance with the stability method (Krippendorff, 1980; Weber, 1985) and a percent agreement (Miles & Huberman, 1994) of greater than 70% was achieved between two coding. In addition, a second coder who is an expert mathematics educator coded a part of the data and the percent agreement was calculated as 87%. Then the researcher and the expert came together to resolve coding discrepancies and discussed the issues until an agreement was reached.

Direct quotations were also given to increase the reliability of the research. They were utilized to support the interpretations in the tables. While interpreting the actions of Samet, it was adequate to give only quotations from Samet. However, interpreting the cases where Samet asked students to explain required quotations of dialogues to reflect classroom environment in detail. In the dialogues, the source of expression (teacher, student) and the expressions have been included. The situations where students talked as a crowded group have been indicated as Stud. (together). Explanations have been included (within the expressions) in square brackets and in italics to describe the current situation of the class. "...” in the dialogue means there were other conversations that were skipped.

Data obtained via survey of function concept was also analysed by using descriptive analysis. In the analysis of the survey, MKT components were considered. The findings were served as secondary ones and used to support primary findings obtained from observations. Therefore, triangulation could be used to ensure the validity of data.

Findings

This section will first present the general findings regarding to the examination of Samet's teaching of the function concept in terms of using mathematical descriptions, explanations and justifications. Then they will be presented separately in detail. Table 2 presents a general picture of Samet's teaching of the function concept according to use of mathematical descriptions, explanations and justifications.

Table 2

Use of Mathematical Descriptions, Explanations and Justifications in Samet's Teaching

Teacher's Action	Sufficiency	Number of cases for using mathematical descriptions	Number of cases for using mathematical explanations	Number of cases for using mathematical justifications
Does himself	Sufficient	54	4	-
	Insufficient	1	2	-
	Needed but Absent	-	4	-
Asks students	(student's response) Sufficient	9	1	-
	(student's response) Insufficient	5	14	1
	Total	69	25	1

Table 2 shows that Samet mostly (in 69 cases) used mathematical descriptions, sometimes (in 25 cases) conducted mathematical explanations and only once (in one case) applied mathematical justification in his teaching of the function concept. Samet usually gave mathematical descriptions by himself and he was sufficient while doing this. When he wanted the students to give descriptions, the students often did it sufficiently, though sometimes they could not. In the case of mathematical explanations, Samet often asked the students to do it and sometimes did it himself. However, the students were insufficient in making mathematical explanations in almost all cases. It was also a remarkable finding that only one case of mathematical justification exists, where a student was asked to provide the justification and was insufficient in doing so.

Findings Related to Mathematical Descriptions

The findings about the examination of Samet's teaching in the line with mathematical descriptions are given via Table 3 in more detail. For presenting findings, a six-column table was used. In the table, the first column indicated the teacher's actions, the second column summarized the sufficiency of the action, the third column showed to which lesson the findings belonged, the fourth column contained the frequency of the action, the fifth column listed the teacher's statement and the sixth

column contained the author's interpretation of what the mathematical description/explanation/justification was. Sub-codes were placed in the rows. Teacher actions were divided into two rows; the first was what he was doing himself, the second showed what he asked the students to do. This distinction was determined depending on who will do it.

Table 3
Use of Mathematical Descriptions in Samet's Teaching

Teacher's Action	Sufficiency	L	n	A Sample Statement	Mathematical description
Does himself	Sufficient	L1	18	<i>Then let's define such a relation, let's define a beta relation, consisting of ordered pairs of x and y.</i>	A relation is made up of ordered pairs.
		L2	10	<i>Now what is the function? It is the special form of the relation, there is a domain set, range set and image set, right?</i>	Function is the special form of the relation.
		L3	8	<i>In the rule of function, e.g. f(1), what did we do? In other words, we were replacing 1 in the rule of function to find the matched element.</i>	We must write that element at the time we see x in the function to find the image under the function f.
		L4	8	<i>When a graph of a function is given, we plot parallel lines parallel to the x-axis to see if the function is one to one.</i>	Description for horizontal test
		L5	10	<i>The function f(x) will only have x, the coefficient of x will have no value other than 1. So, what if we want this function to be a unit function? f(x) = x.</i>	The unit function must be in the form f(x) = x.
	Insufficient	L1	1	<i>If the root degree is double, the absolute value cannot be a negative value. Either zero or a positive value. Why is this said? To specify a real number ...</i>	If the root degree of an absolute value expression is even, inside of the root must always be positive or zero.
	Needed but Absent	--	--		
Asks students to do	(student's response)	L3	3	<i>What we have said is that if a function is different in certain subranges of the domain set, what do we call it?</i>	Desired description for piecewise function
	Sufficient	L4	2	<i>It is not a one to one function, why not? Seda, why not?</i>	Desired description for one-to-one function

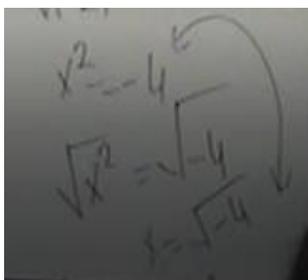
	L5	4	Will the term x be in the constant function?	Desired description for constant function
(student's response) Insufficient	L3	1	What were we doing in this relation? If we say A to B , then on which axis do we write set A , Berke?	Desired description for graph of the relation
	L4	1	Why is the function not one to one?	Desired description for being a one-to-one function
	L5	3	Aykut, how is a function a unit function?	Desired description for unit function

L: Lesson, n: number of cases

If we look at the findings related to the mathematical descriptions that Samet used in his teaching in more detail, it is evident that Samet usually used mathematical descriptions by himself in each lesson and he was sufficient while doing this. For example, Samet made a mathematical description sufficiently for mathematical relations by using the statement “Then let's define such a relation, let's define a beta relation, consisting of ordered pairs of x and y ,” in his first lesson. By this sentence he pointed out that a relation is made up of ordered pairs. In the second lesson, he introduced the function concept through the mathematical relation and emphasized that the function is the special form of the relation.

Samet was insufficient in making mathematical descriptions in one case. In this case, the students were confused and had some questions about if x^2 could be equal to -4 or not. Samet tried to give descriptions in response to these questions by highlighting that if the root degree of an absolute value expression is even, the inside of the root must always be positive or zero. He tried to give this description by the method of finding a contradiction. A screen shot of the board and Samet's statements related to this moment are given in Figure 1.

Figure 1. A screen shot of the board and Samet's statements



Let's go wrong. Let's assume that this is true [writing $x^2=-4$], say something like this. Can I take the root of each side? Ok I took. One more thing, we are making another mistake, we say that when we speak absolute value, if there is no information about x , how do we normally remove the double powers? Absolute x . Let's say we made one more mistake on it, a student mistake, we missed it, we took it out as x . You

said it is not true ($x^2=-4$) at the beginning, but now you say it is $x=\sqrt{-4}$. Isn't there a contradiction? Right? So, what does that mean? If there is no number whose square is negative, when the root degree is double, the absolute value cannot be a negative value. It must be either zero or a positive value.

It was understood from the students' comments and questions that Samet's descriptions were not enough to overcome the confusion of the students. Some of the

students stated that they didn't understand. Then, Samet repeated the same sentences for making clearer the mathematical description. It may be related to the teacher's MKT and in particular his knowledge of content and teaching. A similar one to this finding was obtained from the survey of function concept. First, Samet was asked for the definition of the function, and then he was asked for a new definition to give a student who did not understand the first definition. Samet gave Dirichlet-Bourbaki definition first. However, he was not very successful in this regard when he was asked to make a new definition for the student who did not understand. Because he only gave an example of a relation. He said "*Let the relation has a specific rule. It matches each element of A to its square. In this case, pair as (x, x^2) exists (x^2 is in B set). The relation of these pairs is called a function from A to B.*". It can be said that this definition or example given for a student who does not understand the first definition is not enough. These two supporting findings show that there are some shortcomings in Samet's MKT.

Occasionally, Samet asked his students to make descriptions. The students' responses were sometimes sufficient and sometimes not. When Samet asked a student to make a mathematical description and the student could not make it at once, he continued his teaching and ignored the effort of the student. Instead, he could have helped or encouraged the student to make his/her description in detail. In his fifth lesson, the below dialogue was conducted:

Samet: What did we call the unit function? So anyhow, a function is a unit function. Aykut? Any function is a unit function. It is called a unit function.

Aykut: $f(x)$ will be equal to x , then the value that is given as x will be equal to x .

Samet: Friends, did I not tell you here is the unit function?

Aykut: Yes.

Samet: Now I will ask something for the unit function. For example, if we define a function $f(x)=x^2$, would this unit be a function?

As seen in the dialogue, Samet asked a student (Aykut) a question and wanted a description of the unit function. Aykut actually gave the right answer. However, Samet did not support Aykut's answer, and he continued to play an active role in his teaching. This suggests that it is a formality for the teacher to ask the student questions. When the teacher does not care about the answers from the students, the students may think that their thoughts are not significant for the teacher, so they may not give answer next.

Findings Related to Mathematical Explanations

Findings about the examination of Samet's teaching when giving mathematical explanations are given via Table 4 in detail.

Table 4
Use of Mathematical Explanations in Samet's Teaching

Teacher's Action	Sufficiency	L	n	A Sample Statement	Mathematical explanation
Does himself	Sufficient	L1	4	<i>What we are looking for here is that an element in the domain set must be matched to an element in the set B, that is, in the value set. Ok? This is not a function. You can think of 2a, 2b, 2c. Can you be both in this class and in other class at the same time?</i>	Explanation of the conditions of being a function
	Insufficient	L1	2	<i>So, there is the rule of function. According to the rule, we are doing this matching. We do not do it randomly, okay?</i>	Explanation of "each function must have a rule".
	Needed but Absent	L1	4	<i>Someone may be a mother of 5 children. But is it possible that a child has 5 mothers?</i>	Explanation of the conditions of being a function through a daily life example
Asks students to do	(student's response) Sufficient	L1	1	<i>f2 is not a function. Why not?</i>	Desired explanation of the conditions of being a function
	(student's response) Insufficient	L1	3	<i>What happened this time to our elements? From Seyma to the mother [matches the element in children set to the set of mothers with arrows]. Ok? Is there a difference here, between beta and the inverse of beta?</i>	Desired explanation on relation and its inverse.
		L2	8	<i>Derman, is it a function or not?... You don't think so, why?</i>	Desired explanation of the conditions of being a function
		L3	1	<i>Why does every element have to be used?</i>	Desired explanation of the conditions of being a function
		L4	2	<i>ax plus b is divided by cx plus d. We say the functions in this format are a constant function. Yes, this is memorized knowledge. I'm passing over it. What do you mean by constant? So, what would you know if one of you calls it a constant function?</i>	Desired explanation of constant function

L: Lesson, n: number of cases

Table 4 shows that Samet sometimes made mathematical explanations in his teaching by himself. However, he often asked students to do it. Most often, he made explanations about the conditions of being a function. It may be appropriate to use mathematical explanations frequently in teaching a new concept. In the teaching of the concept of function, especially at the entrance of the subject, he utilized daily life examples. He gave an example of matching “children to mothers” and tried to construct the function concept on this example. Therefore, Samet could explain the concept of function and the conditions of being a function with daily life examples. While making explanations, he was mostly sufficient. For example, he explained that an element of the domain set cannot be matched with more than one element in the value set if this correspondence is a function by these words: *“This is not a function because you can think of 2a, 2b, 2c. Can you be both in this class and in other class at the same time?”*. In addition, he supported his explanation by a daily life example as already mentioned. Here, the daily life example was an analogy that represents students in a class as elements in a set.

In making some of the explanations, Samet was insufficient. For example, he explained that every function must strictly have a rule by saying *“So there is a rule of function. According to the rule, we are doing this matching. We do not do it randomly, okay?”*. It can be said that Samet’s explanation does not coincide with the arbitrariness of a function. The arbitrary nature of functions indicates that functions do not have to be described by any specific expression, follow some regularity, or be described by a graph with any particular shape (Even, 1990). Similarly, Samet mentioned in the survey of function concept that the function is a matching according to a certain rule. This finding showed Samet's inadequate knowledge of specialized content knowledge within the MKT.

In Samet’s teaching, there were some cases of needed but absent in the context of mathematical explanations. These were the cases where a mathematical explanation was needed but Samet did not make it. One of these cases was an expected explanation of the conditions of being a function using daily life examples. In that case, Samet tried to construct the concept of function by using the inverse of the relation. He tried to find out from the students the reverse of the mothers-children correspondence that he gave in the beginning of the lesson. This situation caused confusion in the students' minds who were trying to construct the function concept. Therefore, it seems that it would have been appropriate for Samet to give an explanation at that time. However, Samet only said *“Someone may be a mother of 5 children. But is it possible that a child has 5 mothers?”*. The teacher passed this part very quickly without being sure that the students understood it. That was not enough to get rid of the students’ confusion. Anyway, the comments and questions from the students were an indication of the fact that the concept has not yet been clearly created in their minds. It was also unclear why Samet chose to use the inverse of the relation. He tried to determine the conditions of being a function by examining the inverse together with the relation itself. But this route that he chose was a little complicated for his students.

Another case was regarding an attempt for moving from the daily life example of the function concept to the algebraic form of it. Samet’s expression was as follows: *“We told it (referring to the example of children and their mothers) right away, but we're not talking about it here, okay? So, now let's write a few examples and determine*

if they are functions. Let's say whether the following relations are functions [repeats]. For example, from $f: Z$ to Z , $(x + 1)/2$, let's show it with f . Let $f(x) = (x+1)/2$ be this."

In teaching the function concept, Samet gave the example of children and their mothers first, and then he gave random correspondences between two sets via a Venn diagram. He proceeded with the lesson by using the rule $f(x) = x^2$ as a mathematical example. After giving this example, he asked if the relations given by different algebraic rules represent a function or not. However, it is understood from the students' questions that the concept of function was not clear to the students. In this case, which requires an effective mathematical explanation, the explanation by Samet was not enough. His expression was not even evaluated as a mathematical explanation. It is thought that Samet's attempt to construct the concept of function by using the inverse of the relation had a negative effect on students' learning at the beginning of the lesson, as stated in the previous paragraph, and this confusion continued for a while.

On the other hand, Samet often asked his students to make explanations during his teaching. It was a good attempt to engage students in the teaching process by asking them to make mathematical explanations. However, the students were insufficient in making mathematical explanations in almost all cases (14 of 15 cases). There was only one case where a student made a sufficient mathematical explanation at Samet's request. In this case, Samet asked student to explain why the relation is not a function and the student explained it sufficiently.

As already mentioned, the students were mostly insufficient in making mathematical explanations. Here is an example of these cases:

Samet: What happened this time to our elements? From Seyma to the mother [*matches the element in children set to the set of mothers with arrows*]. Ok? Is there a difference here, between beta and the inverse of beta?

Students: We changed the locations of x and y ...

Samet: Okay, we changed places within the ordered pair. We changed places of x and y within the ordered pair as we write the reverse of a relation. Is that the only difference? Something else?

In the first lesson, Samet examined the differences between the relation and its inverse and asked students to state it. Although the students gave the correct answer, he passed over it quickly and tried to reach the place he had in mind. Another dialogue regarding a similar case is presented:

Samet: First, write the pairs of the function f , the list of elements. Then where do you show us these elements? Show it in the analytical plane; So, draw your graph.

Student: Ok. Every element from A to B must be used. So...

Samet: Why does every element have to be used?

Student: Because it is a function.

Samet: Ha, that it is, there will not be any unmatched elements in the domain set.

As seen in the dialogue in his third lesson, Samet asked a student to write the function by the list method and then to draw its graph on the board. He used the question "why?" to ask the student to explain the conditions of being a function. Like the previous case, he explained the student's response himself instead of giving an opportunity to the student.

Findings Related to Mathematical Justifications

Findings about the examination of Samet's teaching where he used mathematical justifications are given via Table 5 in detail.

Table 5

Use of Mathematical Justifications in Samet's Teaching

Teacher's Action	Sufficiency	L	n	A Sample Statement	Mathematical justification
Does himself	Sufficient	--	--		
	Insufficient	--	--		
	Needed but Absent	--	--		
Asks students to do	(student's response)	--	--		
	Sufficient				
	(student's response) Insufficient	L1	1	<i>So, what will the function be? A relation that matches the elements in A to the elements in B, but what kind of relation? A special relation, then can we say that every relation is a function?</i>	Every relation is not a function, but every function is a relation.

L: Lesson, n: number of cases

There was only one case of mathematical justification in Samet's teaching of the function concept. In this case, he asked the students to justify if a function is always a relation or a relation is always a function.

Samet: So, what will the function be? A relation that matches the elements in A to the elements in B, but what kind of relation? A special relation, then can we say that every relation is a function?

Stu. (together): No.

Stu. (together): Each function is a relation.

Samet: Each function is a relation. Let's repeat that a function is a relation.

As seen in the dialogue, although Samet asked the students to make a mathematical justification, he didn't allow students to do it. He interfered with their answers and passed on quickly. So, the sentence that "Each function is a relation" remained as something for the students to memorize. The students didn't think about or reason through it. Even though it was a very suitable environment for justification, Samet did not use the opportunity well.

Discussion and Conclusion

This study examined a mathematics teacher's teaching function concept in terms of using mathematical descriptions, explanations and justifications to infer his MKT. When Samet's teaching of the function concept was examined in the context of these components as Hill et al. (2008) categorized, mathematical descriptions were seen the

most frequently. This was followed by mathematical explanations, and mathematical justifications were the least seen. Similar to this study's result, Snider (2016) found that teachers' explanations containing superficial reasoning were most common, followed by procedural explanations, mathematical reasoning and finally, problematic explanations.

If we take a closer look at the results of the research, in addition to making mathematical descriptions and explanations by himself, Samet also frequently asked students to make descriptions and explanations. As Leinhardt (2010) stressed, it was appropriate for Samet to include questions like how and why, but Samet did not support the students or encourage them in explaining their answers briefly. It was enough for Samet to get answers from the students to his questions, and he had no intention of learning their underlying thoughts. However, it is an important pedagogical strategy to encourage elaboration of students' responses (Fraivillig, Murphy, & Fuson, 1999). Ball et al. (2008) also pointed to knowledge of content and students as a knowledge domain of MKT. They determined that teachers must be able to hear and interpret students' emerging and incomplete thinking, as expressed in the ways that pupils use language (Ball et al., 2008). NCTM (2000) also supports this claim by emphasizing that effective teaching involves observing students and listening carefully to their ideas and explanations. Moving from the obtained result, it can be said that Samet had some limitations in the frame of knowledge of students.

In some instances, Samet did not make any mathematical explanations although it was necessary. It has been observed that the academic excellence of the students led a quick approach in Samet's teaching. Although Samet attempted to include students in the learning-teaching process, he followed a teacher-centered and traditional teaching approach. Samet's school was at a good level in terms of student achievement. As explained in the "Participant" section, students in Samet's school had higher scores from national tests for entering this school. Therefore, this profile of the school may be an important factor at the result. In the dialogues provided in the above sections, it is seen that Samet received quick and accurate answers from the students to the most of the questions. This rapid progress also created a lack of time devoted to explanations or justifications. There was only one case of justification in Samet's teaching process, but he did not catch the chance to engage students in extended reasoning. Making a description and moving quickly was more suited to Samet's teaching approach. Samet often considered student responses similar to his own explanation he had in mind. This was similar to Ms. Hanes, a teacher who participated in Forman, McCormick and Donato's (1997) research. In addition, the adoption of a different approach for constructing the function concept (constructing the function concept by exploiting the relation and the inverse of relation) made Samet a little distressed. Because this approach confused the students' minds.

Developing sound explanations that justify the steps of the algorithm, and explaining their meaning, involves knowing much more about the algorithm than simply being able to perform it (Ball & Bass, 2002). For this reason, it is important to examine the teacher's knowledge when examining the use of descriptions, explanations and justifications. In this study, a mathematics teacher's (Samet's) knowledge (in the context of MKT) tried to be examined and the results of the study gave the idea that he had some limited knowledge of content and students and he reflected this in his teaching. Similarly, Tataroğlu-Taşdan and Yiğit-Koyunkaya (2017) found that pre-

service mathematics teachers had limited knowledge regarding teaching of function concept and they had difficulties to reflect their knowledge of function concept on their teaching. Furthermore, some deficiencies in knowledge of content and teaching may also have caused insufficient use of descriptions, explanations and justifications. Snider (2016) emphasizes that a good explanation requires more than common content knowledge, because learners are unlikely to understand a mathematical idea in its fully compressed final form. Few findings gave the idea of Samet's deficiencies in the specialized content knowledge, but it is also worth for further examining. The results of Hill et al. (2008) support this argument about Samet's MKT. They found that teachers with stronger MKT responded more appropriately to students and chose examples that helped students construct meaning of the targeted concepts and processes; teachers with weaker MKT were not successful at selecting and sequencing examples, presenting and elaborating upon textbook definitions, and using representations (Charalambous, 2010).

This study focused on a mathematics teacher's teaching of the function concept and tried to contribute the literature on mathematics teachers' MKT in terms of using mathematical descriptions, explanations and justifications. Unlike Hill et al. (2008), the codes were not converted into points, with a rubric to measure the mathematical quality of instruction. This study attempted to conduct an in-depth analysis of one mathematics teacher's use of mathematical descriptions, explanations and justifications for a deeper understanding of his teaching the function concept. The results obtained from this teacher's teaching suggest that mathematics teachers in Turkey might have deficiencies related to their MKT and they might have difficulty in reflecting their existing knowledge to their teaching. In this context, it is suggested that teachers should be supported with professional development programs in order to develop their knowledge and to use this knowledge in practice. The findings provided some suggestions for future studies. Further research could extend the study by including more teachers and investigating the teaching of other mathematical concepts. Multidimensional analysis of data which is collected from different samples using different tools could give different results regarding teachers' MKT. In addition, researchers could study with teachers who have different backgrounds (different years of experience or involved/not involved in professional development programs) and they could examine the effects of these variables on teachers' knowledge and practice.

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